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The heterotic dyonic instanton

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Abstract: The static Yang-Mills-Higgs dyonic instanton is shown to have a non-vanishing, but anti-self-dual, angular momentum 2-form with skew eigenvalues equal to the electric charge; for large charge the angular momentum causes the instanton to expand into a hyper-spherical shell. A class of exact multi dyonic instantons is then found and then generalized to a new class of 1/4 supersymmetric, non-singular, stationary, exact solutions of the ten-dimensional supergravity/Yang-Mills theory. These self-gravitating dyonic instantons yield new heterotic string solitons, to leading order in the inverse string tension.

Keywords: Solitons Monopoles and Instantons, Superstrings and Heterotic Strings, Supergravity Models.

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Contents

1. Introduction

The SU(2) Yang-Mills (YM) instanton can be interpreted as a static 1/2-supersymmetric five-brane of the ten-dimensional ($D = 10$) super-Yang-Mills (SYM) theory [1, 2]. It has a generalisation to a five-brane solution of the effective $D = 10$ supergravity/SYM field theory of the heterotic string, with an instanton core in some SU(2) subgroup of the heterotic gauge group [3]. This 'heterotic five-brane' can also be interpreted as the lift to $D = 10$ of a self-gravitating instantonic-soliton in the $N = 2$ truncation of the $D = 5$ supergravity/SYM theory obtained by dimensional reduction [4], provided that the SU(2) subgroup of the instanton remains unbroken.

The $N = 2$ $D = 5$ supergravity/SYM theory can also be interpreted as the reduction and (consistent) truncation of a (1,0)-supersymmetric supergravity/SYM theory in $D = 6$ (which can itself be obtained from $D = 10$ by reduction and truncation). The 'extra' component of the YM field in $D = 6$ becomes the adjoint Higgs field in $D = 5$ and if this acquires an expectation value the SU(2) gauge group will be spontaneously broken to U(1). In this case, any configuration with non-zero instanton charge is unstable against implosion to a singular field configuration. However, it was recently shown for the flat-space $D = 5$ SYM-Higgs theory that an instanton configuration can be stabilized at a non-zero radius by the inclusion of electric charge [5]; specifically, the energy is minimized by a *static* 1/4 supersymmetric configuration called a 'dyonic instanton' (which has a IIA superstring interpretation as a D0 charge distribution on two parallel D4-branes that is prevented from collapse by a IIA string stretched between the D4-branes [6]).

The dyonic instanton of the $D = 5$ SYM-Higgs theory can also be interpreted as a solution of the pure $D = 6$ SYM theory with the charge of the $D = 5$ solution re-interpreted as momentum in the extra direction. As such, it has an immediate generalisation to a 1/4 supersymmetric solution of the $D = 10$ SYM theory. An obvious question is whether this $D = 10$ SYM configuration can serve as the core of a non-singular self-gravitating 1/4 supersymmetric 'heterotic dyonic instanton' solution of the $D = 10$ supergravity/SYM theory. In addressing this question it is crucial to take into account an unusual feature of the dyonic instanton that has not hitherto been appreciated: although it is a *static* solution of the $D = 5$ SYM-Higgs theory, *it has a non-zero angular momentum, proportional to the electric charge*; inspection of the energy density shows that for large electric charge the angular momentum causes the instanton to expand into a hyper-spherical shell. More precisely, if the instanton is self-dual then the angular momentum 2-form L is anti-self-dual, and vice versa, with skew eigenvalues equal to the electric charge. Although it is a surprise to find that a static soliton may have a non-zero L, one might have anticipated that any non-zero L would be (anti)self-dual because this is known to be a consequence of preservation of supersymmetry in the context of $D = 5$ black holes [7]. In any case, it follows that a generalization of the dyonic instanton to a 1/4 supersymmetric solution of the $D = 10$ supergravity/SYM theory should also have non-zero angular momentum. This means that the solution we seek is not a simple 'superposition' of the 1/2 supersymmetric supergravity/SYM solutions having only instanton charge or only electric charge. However, there is another method available, which we outline, and then exploit, below.

We shall work with the version of the supergravity/SYM theory in which the usual 2-form potential in the supergravity multiplet is replaced by its 6-form dual A with 7-form field strength F [8]. The bosonic action in Einstein frame is

$$\mathcal{S} = \int d^{10}x\,\sqrt{-g}\left(R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2\cdot 7!}e^{\phi}F^2 - \frac{\alpha'}{4}e^{-\phi/2}\,\mathrm{tr}(G^2)\right) + \frac{\alpha'}{2}\int A\wedge\mathrm{tr}(G\wedge G)\,, \tag{1.1}$$

where G is the YM 2-form field strength for the matrix-valued YM gauge field 1-form B. We have chosen to write this action as it would appear (after dualization of the 6-form gauge potential) in an α' expansion of the effective action for the heterotic string; α' has dimensions of length squared and is proportional to the inverse string tension.[1] Note the presence of the AGG Chern-Simons term, which plays an important role in the self-gravitating dyonic instanton solution. In the effective action for the heterotic string there is also a term of the form $\alpha' \int A \wedge R \wedge R$ but this is not needed for supersymmetry and (as in [3]) is actually higher-order in α' for the self-gravitating dyonic instanton. Of course, for heterotic string applications the gauge group is $E_8 \times E_8$ or $\mathrm{Spin}(32)/\mathbb{Z}_2$ but as only an SU(2) subgroup will be needed here we shall take the gauge group to be SU(2).

A dyonic instanton has a definite size, inversely proportional to the square root of the Higgs field expectation value [5]. As a source for the $D = 10$ supergravity fields it may therefore be made very diffuse by choosing a very small expectation value for the Higgs field. For a diffuse source the weak field approximation is good and for this we need only the linearized supergravity equations. We begin by finding the solution of these linearized equations; these linearized results serve to motivate an ansatz for the full solution, which we then refine by demanding that it preserve 1/4 supersymmetry. It is crucial to the success of this step that the supersymmetry transformations of the fermions in the supergravity multiplet be independent of the YM fields. This is true in $D = 10$, but not necessarily in D $<$ 10 because of the field redefinitions needed to put the reduced action in standard form; for this reason the $D = 10$ starting point chosen here is the simplest.

The 1/4-supersymmetric ansatz for the supergravity fields can now be used in the gaugino supersymmetry transformation; setting this to zero and imposing the conditions on the supersymmetry parameter already found from the flat space analysis yields the conditions for preservation of 1/4 supersymmetry in the YM sector. In principle these might depend on the supergravity fields because the gaugino supersymmetry transformation certainly depends on them (even in $D = 10$). If this were the case, the dyonic instanton equations would be modified by functions that are determined by the supergravity field equations, but since these equations involve the YM fields we would then be left with an intractable set of coupled equations. Remarkably, however, the conditions for 1/4 supersymmetry in the YM sector turn out to be *exactly the same as in flat-space* (as is known to happen for 1/2 supersymmetry in the zero charge/momentum case [4]). We provide an exact multi dyonic instanton solution of these equations based on the 't Hooft ansatz, thus generalizing the one-instanton solution of [5]. This constitutes a *known* source for the supergravity equations, which reduce to a set Poisson equations. Remarkably, these equations can also be solved exactly. The solution is asymptotic to a black brane solution describing a rotating superposition of an NS-5-brane and a Brinkmann wave, but with a definite value of the angular momentum that is determined by the 5-brane charge and wave momentum.

[1] Note that all terms in the lagrangian have the same dimension if all supergravity fields are taken to be dimensionless and the YM gauge potential is assigned the dimension of inverse length.

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2. Multi dyonic instantons

We begin with a presentation of an explicit multi-instanton generalization of the flat space dyonic instanton of [5], and a determination of some of its properties including, crucially, its angular momentum 2-form. For convenience we will work with the pure YM theory in the flat $D = 6$ spacetime $\mathbb{E}^{(1,4)} \times S^1$ with coordinates

$$x^\mu = (x^0, x^i, x^5)\,, \qquad (i = 1,2,3,4)\,. \tag{2.1}$$

The YM action is

$$\mathcal{S} = -\frac{1}{4}\int d^6x\ \mathrm{tr}\left(G_{\mu\nu}G^{\mu\nu}\right)\,, \tag{2.2}$$

where G is the 2-form field strength

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu - i[B_\mu, B_\nu] \tag{2.3}$$

for the hermitean 1-form potential B. We take B to be in the fundamental representation of SU(2), so

$$B = B^a\left(\frac{\sigma^a}{2}\right)\,, \tag{2.4}$$

where σ^a $(a = 1,2,3)$ are the three Pauli matrices. The energy momentum tensor is

$$T_{\mu\nu} = \mathrm{tr}\left(G_{\mu\lambda}G_\nu{}^\lambda\right) - \frac{1}{4}\eta_{\mu\nu}\,\mathrm{tr}\left(G_{\lambda\rho}G^{\lambda\rho}\right)\,, \tag{2.5}$$

where $\eta_{\mu\nu}$ is the Minkowski metric. There is also an identically-conserved topological 2-form current density

$$\mathcal{J}^{\mu\nu} = \frac{1}{8}\varepsilon^{\mu\nu\rho\lambda\sigma\kappa}\ \mathrm{tr}\left(G_{\rho\lambda}G_{\sigma\kappa}\right)\,. \tag{2.6}$$

Assuming that all fields are independent of x^5 the energy is

$$E = \frac{1}{2}\int d^4x\left[\mathrm{tr}\left(G_{0i}^2\right) + \mathrm{tr}\left(G_{5i}^2\right) + \frac{1}{2}\mathrm{tr}\left(G_{ij}^2\right) + \mathrm{tr}\,G_{05}^2\right]. \tag{2.7}$$

This is subject to the Gauss law constraint

$$\mathcal{D}_i G_{0i} = 0\,, \tag{2.8}$$

where $\mathcal{D}$ is the gauge-covariant derivative. Following [5], we rewrite E as

$$\begin{aligned} E = {}& \frac{1}{2}\int d^4x\left[\mathrm{tr}\left(G_{0i} - s'G_{5i}\right)^2 + \frac{1}{4}\mathrm{tr}\left(G_{ij} - s\tilde{G}_{ij}\right)^2 + \mathrm{tr}\,G_{05}^2\right] + \\ & + \frac{s}{4}\int d^4x\ \mathrm{tr}\left(G_{ij}\tilde{G}_{ij}\right) + s'\int d^4x\ \mathrm{tr}\left(G_{0i}G_{5i}\right)\,, \end{aligned} \tag{2.9}$$

where $\tilde{G}_{ij}$ is the dual field strength, and s and s' are arbitrary signs. Since B_i is independent of x^5 we have

$$\mathrm{tr}\left(G_{0i}G_{5i}\right) = \partial_i\,\mathrm{tr}\left(B_5 G_{i0}\right)\,. \tag{2.10}$$

It follows that, for fixed asymptotic behaviour of the fields,

$$E \geq \left| \int \mathrm{tr}\,(G \wedge G) \right| + \left| \oint dS_i \,\mathrm{tr}\,(B_5 G_{i0}) \right| , \tag{2.11}$$

with equality when

$$G_{ij} = s \frac{1}{2} \epsilon^{ijkl} G_{kl} \tag{2.12}$$

and

$$G_{5i} = s' G_{0i} \tag{2.13}$$

and

$$G_{05} = 0\,. \tag{2.14}$$

The energy of a solution to these equations is

$$E = 4\pi^2 |I| + |vq|\,, \tag{2.15}$$

where I is the instanton number

$$I = \frac{1}{32\pi^2} \int d^4x \, \varepsilon^{ijk\ell} \,\mathrm{tr}\,(G_{ij} G_{k\ell})\,, \tag{2.16}$$

and q is the electric charge

$$q = \frac{1}{v} \oint dS_i \,\mathrm{tr}\,(B_0 G_{i0})\,. \tag{2.17}$$

The conditions on the YM field strength imply the following relations between the components of the energy-momentum tensor and the topological current density:

$$\begin{aligned} T_{ij} &= 0 \\ T_{5i} - s' T_{0i} &= 0\,, \\ T_{00} - 2s' T_{05} + T_{55} &= 0\,, \\ \mathcal{J}^{5i} + s' \mathcal{J}^{0i} &= 0\,, \\ T_{0i} + s s' \mathcal{J}^{0i} &= 0 \\ T_{00} - T_{55} - 2s \mathcal{J}^{05} &= 0\,. \end{aligned} \tag{2.18}$$

We also note, for future reference, that

$$\int d^4x \mathcal{J}^{05} = 4\pi^2 I\,, \qquad \int d^4x \, T^{05} = -s' |vq|\,. \tag{2.19}$$

The second of these integrals is the momentum in the 5-direction. The other space components of the 6-momentum vanish.

As in [5], we shall seek *time-independent* solutions that minimise the energy for fixed I and q. The equations (2.13) and (2.14) can then be solved by setting

$$B_5 = s' B_0\,. \tag{2.20}$$

Because of the Gauss law constraint, B_0 must satisfy

$$\mathcal{D}^2 B_0 = 0\,. \tag{2.21}$$

Given a solution of (2.12), and the boundary condition that

$$B_0 \to v\left(\frac{\sigma^3}{2}\right) \tag{2.22}$$

for constant v, the equation (2.21) has a unique non-singular solution. We shall consider here only those solutions of (2.12) that can be found via the 't Hooft ansatz. This ansatz makes use of the fact that $\mathbb{E}^4$ admits two commuting sets of complex structures obeying the algebra of the quaternions. For one set, η^a, the corresponding Kähler 2-forms are self-dual; for the other set, $\bar{\eta}^a$, they are anti-self-dual. In cartesian coordinates, for which the metric is δ_{ij}, the 2-forms η^a obey the following identities

$$\begin{aligned} \varepsilon^{abc}\eta^b_{ik}\eta^c_{jl} &= -\delta_{ij}\eta^a_{kl} - \delta_{kl}\eta^a_{ij} + \delta_{il}\eta^a_{kj} + \delta_{kj}\eta^a_{il} \\ \eta^a_{ij}\eta^a_{kl} &= \delta_{ik}\delta_{jl} - \delta_{il}\delta_{kj} + \varepsilon_{ijkl} \\ \eta^a_{ik}\eta^b_{kj} &= -\delta^{ab}\delta_{ij} + \varepsilon^{abc}\eta^c_{ij}\,. \end{aligned} \tag{2.23}$$

The same identities are obeyed by the anti-self-dual 2-forms $\bar{\eta}^a$. The 't Hooft ansatz is

$$B^a_i = \bar{\eta}^{aj}_i \partial_j \log H\,, \tag{2.24}$$

and it yields a self-dual 2-form G if H is harmonic on $\mathbb{E}^4$. Given that H is harmonic we have

$$G^a_{ij} = 2H^{-1}\bar{\eta}^{a\,k}_{[j}\partial_{i]}\partial_k H + 4H^{-2}\left(\bar{\eta}^{ak}_{[i}\partial_{j]}H - \frac{1}{4}\bar{\eta}^a_{ij}\partial_k H\right)\partial_k H \tag{2.25}$$

replacing $\bar{\eta}^a$ by η^a yields an anti-self-dual G. Note that G is self-dual for anti-self-dual Kähler 2-forms and *vice-versa*. If we choose H to have N isolated point singularies, and $H = 1$ at infinity, then we have a non-singular solution with $I = N$. The asymptotic form of this solution is

$$H = 1 + \frac{\rho^2}{r^2} + \mathcal{O}\left(r^{-3}\right), \tag{2.26}$$

where ρ^2 is the sum of the residues of the N singularities. The same ansatz with η in place of $\bar{\eta}$ yields an anti-self-dual G with $I = -N$.

Given B_i of the form (2.24) (and H harmonic on $\mathbb{E}^4$), the equation (2.21) has the solution

$$B_0 = vH^{-1}\left(\frac{\sigma^3}{2}\right). \tag{2.27}$$

When this is used in the formula (2.17) for the electric charge we find that

$$q = 2\pi^2 v\rho^2\,. \tag{2.28}$$

For the one-instanton solution with

$$H = 1 + \frac{\rho^2}{r^2} \tag{2.29}$$

we recover the dyonic instanton with $I = 1$ of [5]. In this case the asymptotic form (2.26) is exact, and ρ can be interpreted as the instanton size. For fixed q we have [5]

$$\rho = \frac{1}{\pi}\sqrt{\frac{q}{2v}}\,, \tag{2.30}$$

so the instanton size is not a free parameter; the collapse of the instanton to zero size is prevented by its charge. An alternative explanation, as we explain below, is that the collapse is prevented by angular momentum.

3. Angular momentum

The angular momentum 2-form (on $\mathbb{E}^4$) is

$$L_{ij} = \int d^4x \left(x^i T_{0j} - x^j T_{0i}\right). \tag{3.1}$$

Noting that

$$T_{0i} = -\partial_j \,\mathrm{tr}\left(B_0 G_{ij}\right), \tag{3.2}$$

and integrating by parts we deduce that

$$L_{ij} = 2\oint dS_k \left[x^j \,\mathrm{tr}\left(B_0 G_{ik}\right) - x^i \,\mathrm{tr}\left(B_0 G_{jk}\right)\right] - 2\int d^4x \;\mathrm{tr}\left(B_0 G_{ij}\right). \tag{3.3}$$

The second term of this expression must vanish because for, say, self-dual G it would have to be proportional to the anti-self-dual $\bar\eta^3_{ij}$, which is impossible unless the constant of proportionality vanishes. This can be verified from the expression

$$\mathrm{tr}\left(B_0 G_{ij}\right) = v\left(\bar\eta^{3\,k}_{[i}\delta^{\ell}_{j]} - \frac{1}{4}\bar\eta^3_{ij}\delta^{k\ell}\right)\partial_k\partial_\ell H^{-1}\,, \tag{3.4}$$

which follows from (2.25). This implies that

$$\mathrm{tr}\left(B_0 G_{ij}\right) \sim \frac{2v\rho^2}{r^4}\bar\eta^3_{ij} + \frac{4v\rho^2}{r^6}\left(\bar\eta^3_{\ell i}x^j - \bar\eta^3_{\ell j}x^i\right)x^\ell + \cdots \tag{3.5}$$

for large r. Using this in the first term of (3.3) we find that

$$L_{ij} = -2\pi^2 v\rho^2\bar\eta^3_{ij}\,, \tag{3.6}$$

which shows that L is anti-self-dual. For anti-self-dual G we must replace $\bar\eta^a$ by η^a, so that L is then self-dual. Comparing with the expression (2.28) we see that

$$L_{ij} = -q\bar\eta^3_{ij}\,, \tag{3.7}$$

which shows that L is non-zero when q is non-zero.

The angular momentum has a dramatic effect on the energy density $\mathcal{E}$. For the multi dyonic instanton one finds that

$$\mathcal{E} \equiv T_{00} = \frac{1}{4}\Box \left\{ H^{-2} \left[(\partial H)^2 + v^2 \right] \right\}, \tag{3.8}$$

where $(\partial H)^2 = \sum_i (\partial_i H)(\partial_i H)$. For H as in (2.29) this reduces to

$$\mathcal{E} = \frac{24\rho^4}{(r^2+\rho^2)^4} \left[1 + \frac{1}{4}(vr)^2 \right]. \tag{3.9}$$

Using (2.30) and defining the dimensionless constant

$$a = \frac{vq}{2\pi^2}, \tag{3.10}$$

we can rewrite the energy density as

$$\mathcal{E}(vr) = 12v^4a^2 \left(\frac{2+(vr)^2}{[(vr)^2+a]^4} \right). \tag{3.11}$$

For $a < 8$ this function has its maximum, and only stationary point, at $r = 0$. For $a > 8$ the stationary point at $r = 0$ is a minimum and there is one other stationary point, a maximum, at

$$vr = \left(\frac{a-8}{3} \right)^{1/2}. \tag{3.12}$$

For large a the maximum occurs for

$$vr \approx \sqrt{\frac{a}{3}} \gg 1, \tag{3.13}$$

while the ratio of $\mathcal{E}$ at the origin to its value at the maximum goes to zero as $1/a$. In other words, *the angular momentum has has blown up the spherical instanton into a hollow hyper-spherical shell.* Note that angular momentum can prevent the collapse of an $(n-1)$-sphere in $\mathbb{E}^n$ only for *even* n (because the angular momentum two-form is necessarily singular when n is odd); here we have an example of this for $n = 4$.

4. Weak field supergravity

We now move to $D = 10$, taking the $D = 10$ Minkowski space coordinates to be

$$x^M = (x^\mu, x^m) \qquad (m = 6,7,8,9). \tag{4.1}$$

The bosonic equations of motion that follow from the action (1.1) are, on setting $\alpha' = 1$,

$$\partial_M \left(\sqrt{-g}\, \partial^M \phi \right) = \frac{1}{2\cdot 7!} e^{\phi} F^2 - \frac{1}{8} e^{-\phi/2}\, \mathrm{tr}(G^2),$$

$$
\begin{aligned}
\partial_{M_1}\left(\sqrt{-g}e^{\phi}F^{M_1\ldots M_7}\right) &= -\frac{1}{8}\epsilon^{M_2\ldots M_7PQRS}\, tr(G_{PQ}G_{RS})\,,\\
\mathcal{D}_M\left(\sqrt{-g}e^{-\phi/2}G^{MN}\right) &= \frac{1}{7!}\epsilon^{M_1\ldots M_7PQN}F_{M_1\ldots M_7}G_{PQ}\,,\\
\mathcal{G}_{MN} &= \frac{1}{2}\left(T^{(\phi)}_{MN}+T^{(F)}_{MN}+T^{(G)}_{MN}\right),
\end{aligned}
\tag{4.2}
$$

where

$$
\begin{aligned}
\mathcal{G}_{MN} &= R_{MN}-\frac{1}{2}g_{MN}R\,,\\
T^{(\phi)}_{MN} &= -\frac{1}{2}g_{MN}(\partial\phi)^2+\partial_M\phi\partial_N\phi\,,\\
T^{(F)}_{MN} &= \frac{1}{6!}e^{\phi}\left(F_{MP_1\ldots P_6}F_N{}^{P_1\ldots P_6}-\frac{1}{14}g_{MN}F^2\right),\\
T^{(G)}_{MN} &= e^{-\phi/2}\left(tr(G_{MP}G_N{}^P)-\frac{1}{4}g_{MN}\,tr(G^2)\right).
\end{aligned}
\tag{4.3}
$$

Given a diffuse YM source we may use a weak field approximation to these equations. We shall begin by considering this case, for which the $D=10$ dilaton and 7-form field strength are small, as is the metric perturbation $h=g-\eta$, where η is the $D=10$ Minkowski metric. Under these circumstances the linearized supergravity equations suffice, and we can ignore any supergravity corrections to the SYM equations. We shall further assume that the SYM fields satisfy the dyonic instanton equations (2.12), (2.13), (2.8) and (2.14).

Given these assumptions, the linearized A field equation implies the equations

$$
\begin{aligned}
\partial_i F_{i056789} &= \frac{s}{4}\,\mathrm{tr}(G_{ij}G_{ij})\,,\\
\partial_i F_{ij06789} &= \frac{ss'}{2}\,\mathrm{tr}(G_{5i}G_{ij})\,,\\
\partial_i F_{ij56789} &= \frac{s}{2}\,\mathrm{tr}(G_{0i}G_{ij})\,.
\end{aligned}
\tag{4.4}
$$

We similarly find from the linearized dilaton equation that

$$
\Box\phi = -\frac{1}{8}\,\mathrm{tr}(G^2_{ij})\,,
\tag{4.5}
$$

and from the linearized Einstein equation (in the de Donder gauge) that

$$
\begin{aligned}
\Box h_{00} &= \frac{1}{2}\,\mathrm{tr}(G^2_{0i})+\frac{1}{32}\,\mathrm{tr}(G^2_{ij})\,, & \Box h_{55} &= \frac{1}{2}\,\mathrm{tr}(G^2_{0i})-\frac{1}{32}\,\mathrm{tr}(G^2_{ij})\,,\\
\Box h_{50} &= \frac{1}{2}s'\,\mathrm{tr}(G^2_{0i})\,, & \Box h_{0i} &= -\frac{1}{2}\,\mathrm{tr}(G_{0j}G_{ij})\,,\\
\Box h_{5i} &= -\frac{1}{2}s'\,\mathrm{tr}(G_{0j}G_{ij})\,, & \Box h_{ij} &= -\frac{3}{32}\delta_{ij}\,\mathrm{tr}(G^2_{kl})\,,\\
\Box h_{mn} &= \frac{1}{32}\delta_{mn}\,\mathrm{tr}(G^2_{ij})\,.
\end{aligned}
\tag{4.6}
$$

For the other components of h the right-hand side is simply zero. It is already possible to deduce that certain components of the metric and 7-form field strength must be non-zero. We also observe that there are some relations between these components. Specifically,

$$F_{ij06789} = s' F_{ij56789} \,, \tag{4.7}$$

and

$$\begin{aligned} h_{0i} - s' h_{5i} &= 0 \,, \\ h_{00} - 2s' h_{50} + h_{55} &= 0 \,, \\ h_{ij} - \frac{3}{2}\delta_{ij}\left(h_{00} - h_{55}\right) &= 0 \,, \\ h_{mn} + \frac{1}{2}\delta_{mn}\left(h_{00} - h_{55}\right) &= 0 \,. \end{aligned} \tag{4.8}$$

In principle, these relations might only hold in the weak field approximation but we shall assume that they hold for the full solution; this assumption can be justified *a posteriori*.

These results motivate an ansatz for the full solution, which we shall present in terms of the inverse 10-metric, written in terms of an inverse vielbein as

$$g^{MN} = \eta^{\underline{AB}} e^M{}_{\underline{A}} e^N{}_{\underline{B}} \,, \tag{4.9}$$

where the underlined indices are the inertial frame indices, indicating transformation properties under the Lorentz group SO(1,9). The linearized analysis suggests the following ansatz for the components of the inverse vielbein $e^M{}_{\underline{A}}$:

$$e^0{}_{\underline{i}} = e^0{}_{\underline{5}} = e^5{}_{\underline{i}} = 0 \tag{4.10}$$

and

$$\begin{aligned} e^0{}_{\underline{0}} &= \sqrt{AB} \,, & e^5{}_{\underline{0}} &= \sqrt{\frac{A}{B}}(B-1) \,, \\ e^5{}_{\underline{5}} &= -\sqrt{\frac{A}{B}} \,, & e^i{}_{\underline{0}} &= e^i{}_{\underline{5}} = -\frac{1}{\sqrt{AB}} E_i \,, \\ e^i{}_{\underline{j}} &= A^{-3/2}\,\delta^i{}_{\underline{j}} \,, & e^m{}_{\underline{n}} &= \sqrt{A}\,\delta^m{}_{\underline{n}} \,, \end{aligned} \tag{4.11}$$

for some functions A and B. The metric then takes the form

$$\begin{aligned} ds^2 = & -A^{-1}(2 - B - A^2E^2)dt^2 + 2s' A^{-1}(B + A^2E^2 - 1)dt dx_5 + \\ & + A^{-1}(B + A^2E^2)dx_5^2 + 2(dt - dx_5)dx_i E_i A^2 + \\ & + A^3 dx^i dx^j \delta_{ij} + A^{-1} dx^m dx^n \delta_{mn} \,, \end{aligned} \tag{4.12}$$

where $E^2 = E \cdot E$. This metric is asymptotically flat if, as $r \to \infty$,

$$A \to 1 \,, \qquad B \to 1 \,, \qquad E_i \to 0 \,. \tag{4.13}$$

5. Supersymmetry

We shall now refine our ansatz for the supergravity fields by insisting that it preserve 1/4 supersymmetry. To do this we must examine the supersymmetry transformations of the fermion fields. These are the gaugino λ, the dilatino χ and the gravitino ψ_M. Their supersymmetry variations are

$$
\begin{aligned}
\delta\lambda &= \frac{1}{2\sqrt{2}} e^{-\phi/4} G_{MN}\Gamma^{MN}\,\epsilon\,,\\
\delta\chi &= -\frac{1}{2\sqrt{2}}\Gamma^M\partial_M\phi\,\epsilon - \frac{1}{4\sqrt{2}\,7!} e^{\phi/2} F_{M_1\ldots M_7}\Gamma^{M_1\ldots M_7}\,\epsilon\,,\\
\delta\psi_M &= D_M\,\epsilon + \frac{1}{2\cdot 8!} e^{\phi/2} F_{M_1\ldots M_7}\left(3\,\Gamma_M{}^{M_1\ldots M_7} - 7\,\delta_M{}^{M_1}\Gamma^{M_2\ldots M_7}\right)\,\epsilon\,,
\end{aligned}
\tag{5.1}
$$

where ϵ is a chiral spinor parameter,

$$
\Gamma^{\underline{0123456789}}\epsilon = \epsilon\,, \tag{5.2}
$$

and $D_M\epsilon$ is its Lorentz covariant derivative,

$$
D_M\epsilon = \left(\partial_M + \frac{1}{4}\omega_{M\underline{AB}}\Gamma^{\underline{AB}}\right)\epsilon\,, \tag{5.3}
$$

where ω is the spin connection. Its non-zero components (for our inverse vielbein ansatz) are

$$
\begin{aligned}
\omega_{5\underline{i5}} &= -\frac{1}{2}B^{-1/2}A^{-3}[A\partial_i B + B\partial_i A + A^2 E\cdot E\partial_i A + -2A^2E_iE\cdot\partial A -\\
&\quad -\frac{1}{2}A^3\partial_i(E\cdot E) - A^3E\cdot\partial E_i]\,,\\
\omega_{i\underline{j5}} &= \omega_{i\underline{j0}} = -\frac{1}{2}B^{-1/2}[\delta_{ij}3E\cdot\partial A - E_i\partial_j A - E_j\partial_i A + A(\partial_i E_j + \partial_j E_i)]\,,\\
\omega_{0\underline{i5}} &= \frac{1}{2}B^{-1/2}A^{-3}[(B-1)\partial_i A - A\partial_i B - 2A^2E_iE\cdot\partial A + A^2E\cdot E\partial_i A -\\
&\quad -\frac{1}{2}A^3\partial(E\cdot E) - A^3E\cdot\partial E_i]\,,\\
\omega_{0\underline{0i}} &= \frac{1}{2}B^{-1/2}A^{-3}[\partial_i A + A\partial_i B - A^2E\cdot E\partial_i A + 2A^2E_iE\cdot A +\\
&\quad +\frac{1}{2}A^3\partial_i(E\cdot E) + A^3E\cdot\partial E_i]\,,\\
\omega_{5\underline{0i}} &= -\frac{1}{2}B^{-1/2}A^{-2}[\partial_i B - AE\cdot E\partial_i A + 2AE_iE\cdot A + \frac{1}{2}A^2\partial_i(E\cdot E) +\\
&\quad +A^2E\cdot\partial E_i]\,,\\
\omega_{5\underline{ij}} &= -\omega_{0\underline{ij}} = A^{-2}\left(-2E_{[i}\partial_{j]}A + A\partial_{[i}E_{j]}\right)\,,\\
\omega_{5\underline{05}} &= -\omega_{0\underline{05}} = \frac{1}{2}A^{-2}E\cdot\partial A\,,
\end{aligned}
$$

$$\begin{aligned}
\omega_{i\underline{05}} &= \frac{1}{2}B^{-1}\partial_i B\,,\\
\omega_{i\underline{jk}} &= \frac{3}{2}A^{-1}\left(\delta_{ij}\partial_k A - \delta_{ik}\partial_j A\right)\,,\\
\omega_{m\underline{in}} &= \frac{1}{2}\delta_{mn}A^{-3}\partial_i A\,,\\
\omega_{m\underline{0n}} = \omega_{m\underline{5n}} &= -\frac{1}{2}B^{-1/2}A^{-2}\delta_{mn}(E\cdot\partial A)\,.
\end{aligned} \tag{5.4}$$

We will also need the gamma-matrix identity

$$\Gamma^{\underline{ij}}\varepsilon^{ijkl} = -2\Gamma^{\underline{1234}}\Gamma^{\underline{kl}}\,. \tag{5.5}$$

We are now in a position to determine the conditions imposed by preservation of 1/4 supersymmetry. The fermion field variations must vanish when the spinor parameter ϵ is subject to the appropriate conditions, which are essentially determined by the vanishing of the gaugino variation. Given our ansatz for the supergravity fields, it can be shown that any solution of the *flat-space* dyonic instanton equations also solves the full supergravity-coupled YM equations. Anticipating this fact, we may easily determine the conditions to be imposed on the ϵ such that $\delta\lambda = 0$. Using the chirality of ϵ and the identity (5.5), one finds preservation of 1/4 supersymmetry provided that

$$\Gamma^{\underline{0}}\epsilon = -\Gamma^{\underline{5}}\epsilon\,, \qquad (1-\Gamma^{\underline{1234}})\epsilon = 0\,. \tag{5.6}$$

Because of the chirality of ϵ, it then follows that

$$\Gamma^{\underline{6789}}\epsilon = \epsilon\,. \tag{5.7}$$

We now turn to the dilatino variation. We will assume (4.7) because this was suggested by the linearized analysis. Using this, and the ansatz for the metric, in $\delta\chi = 0$ we deduce that

$$\Gamma^{\underline{i}}\left(\partial_i\phi + \frac{1}{2}e^{\phi/2}A^3F_{056789i}\Gamma^{\underline{6789}}\right)\epsilon = 0\,. \tag{5.8}$$

This is satisfied (without further conditions on ϵ) if

$$F_{056789i} = 2A^{-3}e^{-\phi/2}\partial_i\phi\,. \tag{5.9}$$

Finally, we must consider the gravitino variation. We will consider the time and space components separately. First, the vanishing of the time-component of $\delta\psi_M$ yields the equation[2]

$$\begin{aligned}
0 = &\left(E_i - B^{1/2}A^{-1}\Gamma^{\underline{i}}\right)\left(\partial_i A - \frac{1}{2}A^4e^{\phi/2}F_{056789i}\right)\epsilon +\\
&+\left(-2E_{[i}\partial_{j]}A + A\partial_{[i}E_{j]} - \frac{3}{2}e^{\phi/2}A^4F_{056789[i}E_{j]} + \frac{1}{2}F_{06789ij}\right)\Gamma^{\underline{ij}}\epsilon\,.
\end{aligned} \tag{5.10}$$

[2]We use conventions such that $2\partial_{[i}V_{j]} = \partial_i V_j - \partial_j V_i$.

Given (5.9), this condition is satisfied (without further constraints on ϵ) if

$$
\begin{aligned}
A &= e^{\phi/2}\,, \\
F_{056789i} &= -\partial_i\left(A^{-4}\right)\,, \\
F_{06789ij} &= -2\partial_{[i}\left(A^{-1}E_{j]}\right)\,,
\end{aligned}
\tag{5.11}
$$

with $F_{ij56789}$ given by (4.7). Note that the restrictions on F are consistent with the Bianchi identity $dF = 0$. We have still to consider the space components of the gravitino variation. It turns out that the 5-component now vanishes identically while the i-components vanish provided that

$$
\partial_i\epsilon + \frac{1}{4}\partial_i \ln(AB)\epsilon = 0\,. \tag{5.12}
$$

This is solved by setting

$$
\epsilon = (AB)^{-1/4}\epsilon_0\,, \tag{5.13}
$$

for a constant chiral spinor ϵ_0 satisfying the same algebraic constraints as ϵ.

To summarize, we see that preservation of 1/4 supersymmetry fixes the dilaton and seven form field strength in terms of the functions A, C and E_i appearing in the metric, but does not constrain these functions. They are constrained by the equations of motion, however, and we now turn to consider them.

6. The multi heterotic dyonic instanton

We now turn to the full field equations. Consider first the YM field equation. Using the metric ansatz and (5.11), one finds that the time component of this equation reduces to the *flat-space* Gauss law constraint (2.8). Moreover, the space components of the YM equation are solved by any solution of the *flat space* equations (2.12) and (2.13). The full YM equations are therefore solved by any solution of the *flat space* dyonic instanton. Given the solution of these equations previously discussed we have a *known* source for the remaining, supergravity, equations. We now turn to these equations, setting

$$
s = 1\,, \qquad s' = -1\,, \tag{6.1}
$$

for convenience. Using the metric ansatz and (5.11) one finds that the dilaton equation reduces to

$$
\Box\left(A^4\right) = -\frac{1}{4}\,\mathrm{tr}(G_{ij}G_{ij})\,. \tag{6.2}
$$

The equation for the supergravity six form gauge field is now identically satisfied except for the $06789k$-component, which yields,

$$
\partial_i\left[\partial_{[i}\left(A^3E_{j]}\right)\right] = -\frac{1}{2}\,\mathrm{tr}(G_{0i}G_{ij})\,, \tag{6.3}
$$

where we have used the self-duality of G_{ij} to simplify the right-hand side.

Finally, we must consider the Einstein equation. As in the linearized analysis we shall impose the de Donder gauge condition

$$\partial_M \left(\sqrt{-\det g}\, g^{MN} \right) = 0\,. \tag{6.4}$$

For our metric this is equivalent to

$$\partial_i (A^3 E_i) = 0\,, \tag{6.5}$$

so in this gauge (6.3) simplifies to

$$\Box (A^3 E_j) = -\operatorname{tr}(G_{0i} G_{ij})\,. \tag{6.6}$$

Note that this equation is consistent with (6.5) since $\operatorname{tr}(G_{0i}G_{ij}) = -\partial_i \operatorname{tr}(B_0 G_{ij})$. The Einstein equations in this gauge reduce to the single equation

$$\Box (B + A^2 E^2) = \operatorname{tr}(G_{0i}^2)\,. \tag{6.7}$$

When these equations for A, B and E_i are written in terms of the new functions

$$f_5 = A^4\,, \qquad f_w = B - 1 + A^2 E^2\,, \qquad f_i = A^3 E_i\,, \tag{6.8}$$

they become the Poisson equations

$$\begin{aligned} \Box f_5 &= -\frac{1}{4} \operatorname{tr}(G_{ij}^2)\,, \\ \Box f_w &= \operatorname{tr}(G_{0i}^2)\,, \\ \Box f_j &= -\operatorname{tr}(G_{0i} G_{ij})\,. \end{aligned} \tag{6.9}$$

For the multi-dyonic instanton configuration presented in section 2, these become

$$\begin{aligned} \Box f_5 &= -\frac{1}{4} \Box \left[H^{-2} (\partial H)^2 \right]\,, \\ \Box f_w &= -\frac{v^2}{4} \Box H^{-2}\,, \\ \Box f_j &= \Box \left(\frac{v}{4} H^{-2} \bar{\eta}^3_{jk} \partial_k H \right)\,, \end{aligned} \tag{6.10}$$

where $(\partial H)^2 = (\partial_i H)(\partial_i H)$, which should be solved subject to the asymptotic boundary conditions

$$f_5 \to 1\,, \qquad f_w \to 0\,. \qquad f_i \to 0\,. \tag{6.11}$$

Given that

$$H = 1 + \sum_\alpha \frac{\rho_\alpha^2}{|x - x_\alpha|^2}\,, \tag{6.12}$$

the equations (6.10) have the following solution

$$
\begin{aligned}
f_5 &= 1 + \alpha' \left[\sum_{i=1}^{N} |x - x_i|^{-2} - \frac{1}{4} H^{-2} (\partial H)^2 \right] , \\
f_w &= \frac{\alpha'}{4} v^2 \left(1 - H^{-2}\right) , \\
f_j &= -\frac{\alpha'}{4} v \bar{\eta}^3_{jk} \partial_k H^{-1} , \qquad (6.13)
\end{aligned}
$$

where we have re-instated the dimensional parameter α'. The sum of poles in the expression for f_5 is needed to ensure non-singularity of the metric at poles of H. Note also that the gauge condition $\partial_i f_i = 0$ is an identity because of the antisymmetry of the matrix $\bar{\eta}^3$. For $N = 1$ this solution reduces to

$$
\begin{aligned}
f_5 &= 1 + \alpha' \frac{(r^2 + 2\rho^2)}{(r^2 + \rho^2)^2} , \\
f_w &= \frac{\alpha'}{2} v^2 \rho^2 \left(\frac{\rho^2 + 2r^2}{(r^2 + \rho^2)^2} \right) , \\
f_j &= -\frac{\alpha'}{2} v \rho^2 \left(r^2 + \rho^2\right)^{-2} \bar{\eta}^3_{jk} x^k . \qquad (6.14)
\end{aligned}
$$

The metric in terms of the functions f_5, f_w and f_i is

$$
\begin{aligned}
ds^2 = f_5^{-\frac{1}{4}} \Big[&- (1 - f_w) dt^2 - 2 f_w dt dx^5 + (1 + f_w)(dx^5)^2 + \\
&+ 2 f_i dx^i (dt - dx^5) \Big] + f_5^{\frac{3}{4}} dx^i dx^j \delta_{ij} + f_5^{-1,4} dx^m dx^n \delta_{mn} , \qquad (6.15)
\end{aligned}
$$

while the other non-zero supergravity fields are

$$
\begin{aligned}
e^{2\phi} &= f_5 , \\
F_{056789i} &= -\partial_i (f_5^{-1}) , \\
F_{06789ij} &= -2 \partial_{[i} (f_{j]} f_5^{-\frac{1}{4}}) . \qquad (6.16)
\end{aligned}
$$

The asymptotic form of the functions f_5, f_w and f_i determining the supergravity fields of the dyonic instanton source are

$$
f_5 = 1 + \frac{\alpha' |I|}{r^2} , \qquad f_w = \frac{\alpha' |vq|}{4\pi^2 r^2} , \qquad f_i = \alpha' \frac{L_{ij} x^j}{4\pi^2 r^4} , \qquad (6.17)
$$

where I is the instanton number, q the charge of the dyonic instanton core and L_{ij} is the angular momentum. This must yield a solution of the pure $D = 10$ supergravity theory. Indeed, it is essentially equivalent[3] to a solution of [9] representing a rotating superposition of an NS-5-brane with a Brinkmann wave. Note, however, that the pure supergravity solution depends on three parameters because the angular momentum is arbitrary. In contrast, the angular momentum is fixed in terms of the other two charges in the non-singular supergravity/YM solution found here.

[3] After correction of a sign error in [9]; we thank Carlos Herdeiro for discussions on this point.

7. ADM energy

One can express the energy of the solution that we have just found in terms of an ADM-type integral over the 3-sphere at transverse spatial infinity, where the metric takes the asymptotic form $g_{MN} = \eta_{MN} + h_{MN}$. In *cartesian* coordinates, this ADM energy is (for $\alpha' = 1$)

$$E = \oint dS_i \left[\partial_j h_{ij} - \partial_i (h_{jj} + \hat{h})\right] , \tag{7.1}$$

where

$$\hat{h} = h_{55} + h_{mm} . \tag{7.2}$$

This differs from the standard ADM formula in that it includes the $\hat{h}$ term that arises from the extra dimensions [10]. Application to the metric of (6.15) yields

$$\begin{aligned} E &= \oint dS \partial_r \left[f_5^{3/4} - \left(4 f_5^{3/4} + f_5^{-1/4}(1 + f_w) + 4 f_d^{-1/4}\right)\right] \\ &= -\oint dS \left(\partial_r f_5 + \partial_r f_w\right) \\ &= 4\pi^2 |I| + |vq| , \end{aligned} \tag{7.3}$$

exactly as in (2.15).

Note that the above result for E is *not* equal to a multiple of the coefficient of h_{00}. The reason for this is as follows. Assuming a weak source, the ADM energy energy can be rewritten as the bulk integral

$$E = \int d^4x \, \partial_i \left[\partial_j h_{ij} - \partial_i (h_{jj} + \hat{h})\right] . \tag{7.4}$$

Use of the linearized version of the Einstein equation $G_{MN} = \frac{1}{2} T_{MN}$ allows us to rewrite this as

$$E = -\frac{1}{7} \int d^4x \left[8 \Box h_{00} + T_{ii} + \hat{T}\right] , \tag{7.5}$$

where

$$\hat{T} = T_{55} + T_{mm} . \tag{7.6}$$

From the observation that

$$\int d^4x \, T_{ii} = \int d^4x \, \partial_j \left(x^i T_{ij}\right) = \oint dS_j \left(x^i T_{ij}\right) , \tag{7.7}$$

we deduce that the integral of T_{ii} vanishes provided that it falls off sufficiently fast near transverse spatial infinity. If it were not for the $\hat{T}$ term we could then recast the integral on the the right hand side of (7.5) as a surface integral involving only h_{00}; this is why the mass of a particle-like object can always be read off from the g_{00} component of the metric. This is not true of p-brane solutions with $p > 0$, as pointed out in [11] for a class of non-extremal brane solutions preserving 1/2 supersymmetry. However, it *is* possible to express the $\hat{T}$ term as a surface integral, with the result that

$$E = -\frac{3}{2}\oint dS_i\,\partial_i(h_{00} - \frac{1}{3}\hat{h})\,. \tag{7.8}$$

Application of this formula to the solution (6.15) again yields the result (7.3).

8. Conclusions

We have found a new exact 1/4 supersymmetric stationary solution of the $D = 10$ supergravity/SYM theory that is asymptotic to a rotating superposition of a black five-brane and a Brinkmann wave of the pure $D = 10$ supergravity theory. However, the interior is 'filled in' with a YM multi dyonic instanton core in such a way that the spacetime topology is the same as that of $D = 10$ Minkowski space (or a toroidal compactification of it), without singularities. We have called this new solution the 'heterotic dyonic instanton' because it generalizes the 1/2 supersymmetric heterotic five-brane solution of [3], which was proposed as a solution of the heterotic string to leading order in an α' expansion. The generalization found here is two-fold: firstly, we have found an exact self-gravitating 5-brane solution with a *multi-instanton* core and, secondly, we have generalized this to the multi dyonic instanton. Of course, the $D = 10$ supergravity/SYM theory serves equally as an effective field theory for the type-I superstring, and our solution can therefore be viewed as an approximate solution to either the heterotic or the type-I superstring.

We have also uncovered some unusual features of the flat-space dyonic instanton, notably that it carries angular momentum despite the fact that all fields are time-independent, and that the instanton expands to a hyper-spherical shell for large angular momentum (corresponding to large electric charge). Because of this angular momentum, the self-gravitating dyonic instanton is not a static solution of the supergravity/SYM equations but rather a *stationary* one. For the 'intersecting black brane' solution of pure supergravity to which this solution is asymptotic the angular momentum is a free parameter, and may be set to zero. In contrast, the angular momentum of the self-gravitating dyonic instanton is not a free parameter, and it is necessarily non-zero. Thus, *the requirement that the core of the supergravity solution be 'filled in' by a non-singular YM configuration fixes an otherwise free parameter.*

One surprise, although not one without precedent for simpler 1/2 supersymmetric supergravity/SYM solutions, is that the *flat space* dyonic instanton solution continues to solve the SYM equations even after coupling to supergravity. This feature means that the Laplace equations that one must solve to find the intersecting black brane solution with the same asymptotic behaviour are replaced by Poisson equations with solutions that exist and are uniquely determined (given the requirement of non-singularity) by this asymptotic behaviour. The coupling to gravity thereby preserves both the solution and its BPS nature in the simplest way imaginable. This may have implications for brane world scenarios because it provides strong evidence that BPS configurations that are initially defined on a brane, without gravity, will survive the coupling to gravity. It would be interesting to see whether a general statement along these lines could be proved.

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